

# Spontaneous Symmetry Breaking in the Brane-World

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A simplified Randall-Sundrum-like model in 6 dimensions is discussed. The extra two dimensions correspond to the cone. The effective four-dimensional scalar self-interacting theory is studied at one-loop level. The contributions due to 6-dimensional parameters in four-dimensional beta-functions appear. Using such beta-functions the one-loop effective potential is calculated. The possibility of spontaneous symmetry breaking due to extra dimensions is demonstrated.

## 1. Introduction

The spontaneous symmetry breaking effect in an external gravitational field has a lot of very interesting characteristics. In particular it is important to remark that the interaction with the external gravitational field may lead to the spontaneous symmetry breaking (see papers [6, 7]). It is possible to give masses for gauge fields without the introduction of a “negative square” mass in the scalar sector. One way of doing this is for example to introduce the term  $\xi R$ , which gives the non-minimal coupling between the scalar and the gravitational fields ([8, 6, 7]).

The influence of the quantum corrections on the spontaneous symmetry breaking is easier to study on the basis of effective potential (EP). Approximate expressions for EP in an external gravitational field of the special form have been obtained in papers (see for example [9, 10, 11, 12]), for the theory of the scalar field with interaction  $\lambda\phi^4$ , scalar electrodynamics and gauge theory with scalars.

In this paper we consider the theory  $\lambda\phi^4$  with mass in a six-dimensional space-time. For this theory we are interesting in the influence of the extra two dimensions on the spontaneous symmetry breaking in the effective four-dimensional theory.

An interesting way to do this is to consider the Randall-Sundrum brane-world models [1], which at the present time already has attracted a great deal of interest in particle physics and phenomenology because of the possibility to resolve the hierarchy problem in a quite natural way.

In a typical approach to modern quantum high energy theory it is necessary to consider quantum fields on a higher dimensional manifold (the bulk) in the presence of extended defects (the boundaries). On the bulk manifold, as well as on the brane, there exist divergences which result in the running of coupling constants in the

standard way. However, it has been known for some time that for spacetimes with boundaries there are not only the usual volume coupling constants but also surface ones [2]. It is known that they influence each other; for example volume interactions are reflected in surface terms, etc [3]. But the situation could be complicated in some brane-world models. For example one may wonder how the running of the bulk couplings influences the running on the brane and vice-versa. It has been demonstrated in ref.[5] that bulk contribution may completely change the standard running behaviour of brane couplings as the effect of bulk couplings. Moreover one may wonder how this influences the spontaneous symmetry breaking. This is the purpose of this work to study the spontaneous symmetry breaking in the model of ref.[5].

## 2. Description of the Model

Our model is given as a massive Euclidean self-interacting scalar in a 6-dimensional space with a conical singularity, due to the presence of 3-brane. The metric is chosen to be [5]

$$ds_6^2 = dr^2 + r^2 d\theta^2 + ds_0^2, \quad (2.1)$$

where  $ds_0^2$  is the 4-dimensional flat metric, the brane is located at  $r = 0$ , and  $\theta$  has a period  $\beta$ ,  $\beta$  being the deficit angle of the cone. When  $\beta = 2\pi/N$ ,  $N$  a positive integer, one is dealing with a less singular manifold, namely an orbifold, while for  $N = 1$ ,  $\beta = 2\pi$ , one has the smooth 2-dimensional plane. The action reads

$$S = \int d^6x \sqrt{g} \left[ \phi \frac{1}{2} (-\square_6 + m^2) \phi + V(\phi) \right] + \int d^4x W(\phi), \quad (2.2)$$

where  $V(\phi) = \frac{g}{4!} \phi^4 + \dots$  denotes a series of scalar bulk couplings. We also introduce a “surface” term which

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depends on surface scalar couplings

$$W(\phi) = \left[ \lambda_0 + \frac{\lambda_2}{2}\phi^2 + \frac{\lambda_4}{4!}\phi^4 + \dots \right], \quad (2.3)$$

namely it may contain a brane tension  $\lambda_0$ , a brane mass  $\lambda_2$ , a  $\phi^4$  coupling  $\lambda_4$ , as well as higher terms. As it has been demonstrated in ref.[5], these surface terms are necessary because we are dealing with a manifold with a conical singularity. We also assume that the brane is not dynamical, namely we are dealing with a rigid brane, and therefore we neglect the brane kinetic term. This finishes our description of the model, for more details one may consult in [5].

### 3. One-loop effective potential

The one-loop correction is determined by the total one-loop fluctuation operator, which reads

$$L_6 = -\square_\beta - \square_4 + M^2 + W''(\Phi)\delta^{(2)}(x), \quad (3.4)$$

where  $\square_\beta$  is the 2-dimensional Laplacian on the cone,  $\Phi$  is the background field and  $M^2 = m^2 + V''(\Phi)$  is an effective mass.

We shall make use of zeta-function regularization and related heat-kernel techniques (see, for example Ref. [13]). Within the one-loop approximation, one needs to evaluate the zeta-function at zero, namely  $\zeta(0|L_6)$ , since this quantity gives rise to the one-loop divergences and governs the one-loop beta functions. There are also contributions due to the conical singularity and the brane delta-function contribution, which gives additive contributions to  $\zeta(0|L_6)$ , which have been diagrammatically evaluated in Ref. [4]. We will take the final results for the calculation of four-dimensional effective beta-functions in refs.[4, 5].

In our theory we are taking in account that four-dimensional curvature  $R = 0$  and  $\phi = \text{const}$ . For this case theory maybe considered as one-loop renormalizable and the technique of refs.[9, 10] to find the one-loop effective potential maybe used. The one-loop RGE equation for potential is

$$DL_{eff} = 0 \quad (3.5)$$

here  $D$  and  $L_{eff}$  are defined in [9] as

$$D = \mu \frac{\partial}{\partial \mu} + \beta_\lambda \frac{\partial}{\partial \lambda} + \gamma \frac{\partial}{\partial m^2} + \gamma_\phi \frac{\partial}{\partial \phi} \quad (3.6)$$

$$L_{eff} = -a(\lambda_4 + E_{\lambda_4}t)\phi^4 - b^2(\lambda_2 + E_m t)\phi^2 \quad (3.7)$$

In our situation  $E_{\lambda_4} = \beta_{\lambda_4}$  and  $E_\gamma = \gamma$ , where the  $\beta_{\lambda_4}$  and  $\gamma$  functions derived in [5] have the form

$$\gamma = \frac{\lambda_2^2}{\pi} + \frac{m^2 g_4}{128\pi^2} - \frac{m^4 \lambda_4}{64\pi^3} \quad (3.8)$$

$$\beta_{\lambda_4} = \frac{4\lambda_2 \lambda_4}{\pi} + \frac{3g_4^2}{128\pi^2} - \frac{m^4 \lambda_6}{64\pi^3} \quad (3.9)$$

We add to equations (3.7) the renormalization conditions

$$\left. \frac{\partial^4 L_{eff}}{\partial \phi^4} \right|_{\phi=\phi_{\lambda_4}} = -4!a\lambda_4, \quad \left. \frac{\partial^2 L_{eff}}{\partial \phi^2} \right|_{\phi=\phi_m} = -2b\lambda_2 \quad (3.10)$$

Using these conditions in (3.7) one finally obtains for  $L_{eff}$  [9, 10]

$$L_{eff} = -\frac{1}{4!}[\lambda_4 + \frac{1}{2}\beta_{\lambda_4} \ln \frac{\tau}{m_{\lambda_4}^2}]\phi^4 - \frac{1}{2}[\lambda_2 + \frac{1}{2}\gamma \ln \frac{\tau}{m_m^2}]\phi^2 \quad (3.11)$$

Here

$$\tau = \lambda_2 + \frac{\lambda_4}{2}\phi^2, \quad m_{\lambda_4}^2 = \lambda_2 + \frac{\lambda_4}{2}\phi_{\lambda_4}^2, \quad m_m^2 = \lambda_2 + \frac{\lambda_4}{2}\phi_m^2 \quad (3.12)$$

This finishes the calculation of one-loop effective potential.

### 4. Spontaneous symmetry breaking

In a flat space with  $\phi = \text{const}$  the existence of an absolute minimum of the EP where EP takes negative value may lead to spontaneous symmetry breaking. One can find the critical value of  $\phi$  from the conditions

$$\frac{\partial V_{eff}}{\partial \phi} = 0, \quad \frac{\partial^2 V_{eff}}{\partial \phi^2} > 0 \quad (4.13)$$

The EP is given by (3.11) and it has the form

$$V_{eff} = \frac{1}{4!}[\lambda_4 + \frac{1}{2}\beta_{\lambda_4} \ln \frac{\tau}{m_{\lambda_4}^2}]\phi^4 + \frac{1}{2}[\lambda_2 + \frac{1}{2}\gamma \ln \frac{\tau}{m_m^2}]\phi^2 \quad (4.14)$$

In order to study the spontaneous symmetry breaking one may consider several cases for the relations between the parameters  $\lambda_2$ ,  $\lambda_4$ ,  $\lambda_6$ ,  $g_4$ , etc and obtain an approximation for the EP. Using the conditions (4.13) it is possible to see if there is spontaneous symmetry breaking for each of the cases. As an example, let us consider that  $\lambda_2 \ll \frac{\lambda_4}{2}\phi^2$  for all of the values for  $\phi$ . It is not difficult to see that in such case it follows that

$$V_{eff} = \frac{1}{4!}\lambda_4\phi^4 + \frac{1}{2}\lambda_2\phi^2 + [\frac{1}{48}\beta_{\lambda_4} \ln \frac{\phi^2}{\phi_{\lambda_4}^2}]\phi^4 + \frac{1}{4}\gamma \ln \frac{\phi^2}{\phi_m^2} \phi^2 \quad (4.15)$$

Now for simplicity we will consider that  $\phi_{\lambda_4} = \phi_m = \phi_o$ . Introducing the dimensionless variable  $x = \frac{\phi^2}{\phi_o^2}$  we may write the EP in the following form

$$V_{eff} = \frac{1}{4!}[\lambda_4 + \frac{1}{2}\beta_{\lambda_4} \ln x]\phi_o^4 x^2 + \frac{1}{2}[\lambda_2 + \frac{1}{2}\gamma \ln x]\phi_o^2 x, \quad (4.16)$$

Finally for calculation it is better to write this expression as

$$\frac{V_{eff}}{\phi_o^4} = \frac{1}{4!}\lambda_4 x^2 + \frac{1}{2}\frac{\lambda_2}{\phi_o^2}x + \left(\frac{1}{48}\beta_{\lambda_4}x \ln x + \frac{\gamma}{4\phi_o^2} \ln x\right)x \quad (4.17)$$

The first derivative for (4.17) is

$$\frac{\partial}{\partial x}\left(\frac{V_{eff}}{\phi_o^4}\right) = \left(\frac{1}{6}\lambda_4 + \frac{1}{48}\beta_{\lambda_4}\right)x + \left(\frac{1}{24}\beta_{\lambda_4}x + \frac{\gamma}{4\phi_o^2}\right) \ln x + \left(\frac{1}{2}\frac{\lambda_2}{\phi_o^2} + \frac{\gamma}{4\phi_o^2}\right) \quad (4.18)$$

Now let us write the first expression in (4.13) when  $x$  takes values around 1. Thus doing  $x = 1 + z$ , where  $z \ll 1$ , we obtain the equation

$$\frac{1}{24}\beta_{\lambda_4}z^2 + \left(\frac{1}{72}\beta_{\lambda_4} + \frac{\gamma}{4}\phi_o^2 + \frac{1}{6\lambda_4}\right)z + \left(\frac{1}{48}\beta_{\lambda_4} + \frac{1}{6}\lambda_4 + \frac{1}{2}\frac{\lambda_2}{\phi_o^2} + \frac{\gamma}{4\phi_o^2}\right) = 0 \quad (4.19)$$

Before solving this equation it is recommended to take the second derivative of (4.17), in order to verify then the second condition for the spontaneous symmetry breaking. Thus we have

$$\frac{\partial^2}{\partial x^2}\left(\frac{V_{eff}}{\phi_o^4}\right) = \frac{1}{6}\lambda_4 + \frac{1}{24}\beta_{\lambda_4} + \frac{1}{24}\beta_{\lambda_4} \ln x + \frac{\gamma}{4\phi_o^2} \frac{1}{x} + \frac{1}{48}\beta_{\lambda_4} \quad (4.20)$$

Solving for  $z$  the equation (4.19) we will use the approximation

$$\beta_{\lambda_4} \approx -\frac{m^4\lambda_6}{64\pi^3}, \quad \gamma \approx \frac{m^2g_4}{128\pi^2} \quad \text{and} \quad m^2g_4 \gg m^4\lambda_6$$

This approximation leads to two possible solutions for  $z$  in (4.19).

The first solution is  $z = 0$  and this value gives us  $x = 1$ . It is not difficult to see that in this case the second derivative (4.20) satisfies the second condition for the spontaneous symmetry breaking.

The second solution

$$z = \frac{3\pi}{2\phi_o^2} \frac{m^2g_4}{m^4\lambda_4}$$

gives us a big value of  $x$  and it is easy to see that the second derivative will be negative, so with this solution there is no spontaneous symmetry breaking.

Now let us consider that  $x \approx 0$ . The equation for the first derivative is written now in the form

$$\frac{\gamma}{4\phi_o^2} \ln x + \left(\frac{1}{2}\frac{\lambda_2}{\phi_o^2} + \frac{\gamma}{4\phi_o^2}\right) = 0 \quad (4.21)$$

The solution for this equation can be written as

$$x = e^{-(1+\frac{2\lambda_2}{\gamma})} \quad (4.22)$$

This value of  $x$  with the approximations that were used before leads to spontaneous symmetry breaking too.

Finally we may say that using the same approximation for the situation when  $x \gg 1$ , it is easy to find an absolute minimum of the EP, which indicates that there is spontaneous symmetry breaking in this case too.

Thus, we demonstrated that there maybe spontaneous symmetry breaking induced by extra two dimensions (in our case chosen as a cone). It occurs mainly due to parameters of higher dimensional theory. That indicates that extra dimensions in brane-worlds may play an important role in phenomena like Higgs effect and spontaneous symmetry breaking. In curved four-dimensional space one expects that there maybe phase transitions induced by extra dimensions what could find various applications in early universe cosmology.

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